Normalized Solutions for a Schrödinger-Bopp-Podolsky System

Danilo Gregorin Afonso (danilo.afonso.g@gmail.com)
Instituto de Matemática e Estatística
Universidade de São Paulo

Abstract. Schrödinger-Poisson systems first appeared as the coupling of the Schrödinger's equation with a newtonian gravitational potential. Benci and Fortunato [1] studied a system of similar nature arising from the coupling of Schrödinger's equation with Maxwell's equations and, under Dirichlet boundary conditions, developed a now standard method to deal with this kind of problem. Later, Pisani and Siciliano [2] treated a Neumann problem for a similar system, where the procedure of Benci and Fortunato fails. In the present work we treat a modification of the problem studied in [2] which consists in the addition of a biharmonic term in the second equation (and imposition of appropriate boundary conditions):

$$
\begin{aligned}
-\Delta u + q\varphi u &= \omega u \\
-\Delta\varphi + \Delta^2\varphi &= qu^2
\end{aligned}
$$

in a smooth, bounded domain Ω subject to

$$
u = 0, \qquad \frac{\partial\varphi}{\partial n} = h_1, \qquad \frac{\partial\Delta\varphi}{\partial n} = h_2
$$

on $\partial\Omega$, where h_1, h_2 are continuous, $q \in C(\overline{\Omega})$, $\omega \in \mathbb{R}$ and $\int_\Omega u^2\, dx = 1$. Motivation for the inclusion of the biharmonic operator comes from the Bopp-Podolsky electrodynamics, where the equation for the electric potential is of this form. We introduce a modification to the problem in order to deal with homogeneous boundary datum, outline the procedure by Benci and Fortunato and point out why it fails, present a technique to overcome this obstacle and prove the existence of infinitely many solutions $(u_n, \omega_n, \varphi_n) \in H_0^1(\Omega) \times \mathbb{R} \times H^2(\Omega)$.

References

[1] Vieri Benci and Donato Fortunato. *An eigenvalue problem for the Schrödinger-Maxwell equations.* Topological Methods in Nonlinear Analysis, 1998.

[2] Lorenzo Pisani and Gaetano Siciliano. *Constrained Schrödinger-Poisson System with Non-Constant Interaction.* Communications in Contemporary Mathematics, Vol 15, 2013.